

MAY 14 1947

641213
-125
RB No. 3J30

~~Copy~~
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WARTIME REPORT

ORIGINALY ISSUED
~~October 1943~~
Restricted Bulletin 3J30

USE OF STAGNATION TEMPERATURE IN CALCULATING RATE OF
HEAT TRANSFER IN AIRCRAFT HEAT EXCHANGERS

By George P. Wood

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

NACA

WASHINGTON

NACA WARTIME REPORTS are reprints of papers originally issued to provide rapid distribution of advance research results to an authorized group requiring them for the war effort. They were previously held under a security status but are now unclassified. Some of these reports were not technically edited. All have been reproduced without change in order to expedite general distribution.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESTRICTED BULLETIN

USE OF STAGNATION TEMPERATURE IN CALCULATING RATE OF
HEAT TRANSFER IN AIRCRAFT HEAT EXCHANGERS

By George P. Wood

SUMMARY

Theoretical and experimental investigations of the effect of frictional heat on the rate of heat transmission at high fluid velocities are briefly reviewed. On the basis of these investigations, calculations are made which show that the use of the stagnation temperature of the cooling air as the effective temperature for heat transfer in an aircraft heat exchanger is sufficiently accurate.

INTRODUCTION

No new analyses or experiments are given in the present paper. Its purpose is to point out that, on the basis of existing information, the proper cooling-air temperature for use in aircraft-heat-exchanger calculations is, to a sufficient degree of accuracy, the stagnation temperature of the cooling air.

At the present time some uncertainty exists concerning the proper cooling-air temperature to be used in calculations of rate of heat transfer when the velocity of the cooling air across the heat-transfer surface is high, or when the velocity is low but the air has been slowed down from a high-velocity air stream. Conventional heat-transfer coefficients generally have been determined from measurements made at rather low air velocities. At high velocities, the heat generated in the boundary layer by friction may be an important factor in the rate at which heat is transferred across the boundary layer from the surface to the fluid. The effect of the frictional heat, which is negligible at low air velocities, is not included in the conventional low-velocity coefficients of heat transfer or in the

conventional definition of the temperature difference between the surface and the fluid. The problem of finding the equation for rate of heat transfer that properly takes into account the conditions in the boundary layer for high fluid velocities has been undertaken by a number of investigators. Because the mathematics of the problem is complex and simplifying assumptions must be made in order to obtain solutions, rigorous results for all types of flow have not yet been obtained. Because many different equations have been derived, some confusion exists. Actually, when typical numerical values are substituted in the various equations, the results, though differing slightly in numerical value, are all of the same order of magnitude. The present paper shows that, for aircraft heat exchangers, a rigorous equation for high-velocity heat transfer is not necessary but that, on the basis of the investigations reviewed, sufficient accuracy results from the use of the stagnation temperature as the effective temperature of the cooling air.

SYMBOLS

c_p	specific heat of fluid at constant pressure, Btu per pound per $^{\circ}\text{F}$
f	friction factor, dimensionless
g	acceleration due to gravity, feet per second per second
h	coefficient of heat transfer for low fluid velocities, Btu per second per square foot per $^{\circ}\text{F}$
H	rate of heat transfer, Btu per second
J	mechanical equivalent of heat (778 ft-lb/Btu)
k	thermal conductivity of fluid, Btu per second per square foot per $^{\circ}\text{F}$ per foot
P	rate of doing work against friction, foot-pounds per second

Pr	Prandtl number, dimensionless, $(c_p \mu g/k)$
S	surface area for heat transfer, square feet
T	temperature, $^{\circ}F$
ΔT	temperature difference, $^{\circ}F$
T_o	free-stream temperature of fluid, $^{\circ}F$
$(\Delta T)_{ad}$	adiabatic temperature change due to compression, $^{\circ}F$
T_{eff}	fluid temperature effective for heat transfer, $^{\circ}F$
$(\Delta T)_{eff}$	difference between T_{eff} and T_f , $^{\circ}F$
T_f	actual temperature of fluid outside boundary layer, $^{\circ}F$
T_{en}	temperature of fluid at heat-exchanger entrance, $^{\circ}F$
T_{ex}	temperature of fluid at heat-exchanger exit, $^{\circ}F$
T_s	stagnation temperature of fluid, $^{\circ}F$
T_{sen}	stagnation temperature of fluid at heat-exchanger entrance, $^{\circ}F$
T_{sex}	stagnation temperature of fluid at heat-exchanger exit, $^{\circ}F$
T_w	temperature of wall, $^{\circ}F$
V	velocity of fluid, feet per second
V_o	free-stream velocity of fluid, feet per second
V_{en}	velocity of fluid at heat-exchanger entrance, feet per second
V_{ex}	velocity of fluid at heat-exchanger exit, feet per second
W	weight rate of flow of fluid, pounds per second
y	normal distance from flat plate, feet

μ	coefficient of viscosity of fluid, slugs per foot-second
ρ	mass density of fluid, slugs per cubic foot
τ	frictional resistance per unit surface area, pounds per square foot

DISCUSSION

Consider a flat plate with its axis along the flow direction. The rate at which heat is transferred between the plate and the fluid is determined by the temperature gradient at the plate in the boundary layer of the fluid and by the thermal conductivity of the fluid; that is,

$$\frac{H}{S} = k \left(\frac{\partial T}{\partial y} \right)_{y=0}$$

The method of determining, from the differential equations of the boundary layer, the effect of the frictional heat on the temperature gradient need not be discussed in the present paper. An idea, however, of the reason that the frictional heat may be an important factor at high speeds can be had from the following simple analysis:

The frictional resistance per unit surface area is the product of friction factor and dynamic pressure

$$\tau = f \frac{\rho V^2}{2}$$

The rate at which frictional work is done per unit surface area is

$$\begin{aligned} \frac{P}{S} &= \tau V \\ &= \frac{f \rho V^3}{2} \end{aligned}$$

The rate of heat transfer per unit surface area is

$$\frac{H}{S} = h \Delta T$$

The ratio of work done to heat transferred is then

$$\frac{P}{H} = \frac{f \rho v^3}{2h \Delta T}$$

By using the relation discussed in reference 1 for correlating friction and heat transfer

$$2h = \frac{c_p g \rho v f}{(Pr)^{2/3}}$$

the ratio between frictional work and transferred heat is obtained as

$$\frac{P}{H} = \frac{v^2 (Pr)^{2/3}}{c_p g \Delta T} \quad (1)$$

The ratio of the heat generated in overcoming friction to the heat transferred between surface and fluid is thus proportional to the square of the fluid velocity. It is therefore to be expected that the effect of the frictional heat is most pronounced at high velocities.

The problem of obtaining a rigorous expression for rate of heat transfer with high fluid velocities has been the subject of a number of investigations. Some of the results that have been obtained are briefly discussed.

For laminar flow, analyses (references 2 and 3) show that the proper temperature difference for use in calculating rate of heat transfer for high velocities and with use of the usual low-velocity coefficients is the difference between the wall temperature T_w and the effective temperature T_{eff} , where T_{eff} is a function of

the Prandtl number of the fluid, the fluid temperature outside the boundary layer T_f , and the increase in temperature that would result from complete adiabatic arrest of the flow outside the boundary layer $(\Delta T)_{ad}$. The quantity

$$(\Delta T)_{eff} = T_{eff} - T_f$$

can be found from figure 1, which is taken from figure 2 of reference 2.

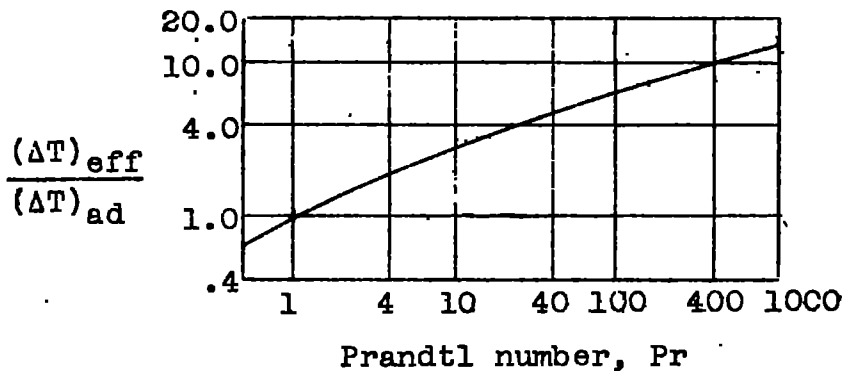


Figure 1.- Ratio $(\Delta T)_{eff}/(\Delta T)_{ad}$ as a function of Prandtl number for laminar flow. (From fig. 2 of reference 2.)

For air, the Prandtl number of which is about 0.75, the value of $(\Delta T)_{eff}/(\Delta T)_{ad}$ is approximately 0.85. The rate of heat transfer between an element of wall area S and air in laminar flow is then calculated by the equation

$$\begin{aligned} \frac{H}{S} &= h(T_w - T_{eff}) \\ &= h[T_w - T_f - (\Delta T)_{eff}] \\ &= h[T_w - T_f - 0.85(\Delta T)_{ad}] \end{aligned}$$

For a given fluid, $(\Delta T)_{ad}$ (in °F) is a function of only velocity. For air,

$$(\Delta T)_{ad} = (0.832 \times 10^{-4}) v^2$$

That the temperature T_{eff} produced at the wall by friction should be a function of the Prandtl number Pr of the fluid seems reasonable because the rate at which heat is generated in the boundary layer is proportional to the coefficient of viscosity of the fluid μ , inasmuch as

$$\tau = \mu \left(\frac{\partial v}{\partial y} \right)_{y=0}$$

and because the rate at which heat is conducted across the boundary layer is proportional to the thermal conductivity of the fluid k .

Experiments made with air flowing past unheated surfaces, which are reported in references 4 and 5, have corroborated the relation

$$(\Delta T)_{eff} = 0.85(\Delta T)_{ad}$$

for laminar flow. (See fig. 2 which is taken from fig. 4 of reference 4.)

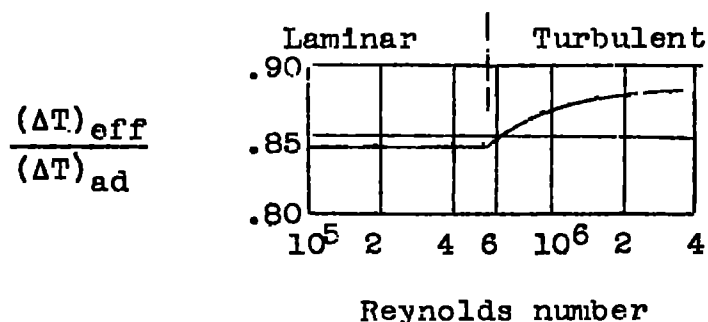


Figure 2.- Ratio $(\Delta T)_{eff}/(\Delta T)_{ad}$ as a function of Reynolds number. Axial wire in flow of air through nozzle. (From fig. 4 of reference 4.)

For turbulent flow, uncertainty concerning the exact distribution of velocity in the laminar sublayer at the fluid boundary and concerning conditions at the interface between the laminar sublayer and the buffer layer, together with the difficulty of obtaining a solution to the differential equations of the boundary layer, has led to somewhat different results in different investigations. Reference 6 gives as the effective fluid temperature T_{eff} the full stagnation temperature, with rate of heat transfer calculated by the equation

$$\begin{aligned}\frac{H}{S} &= h(T_w - T_{eff}) \\ &= h[T_w - T_f - (\Delta T)_{ad}]\end{aligned}$$

The result, however, is not rigorous because the laminar sublayer is not taken into account. In reference 7 another relation is derived but, according to a statement in reference 8, the analysis is based on relatively rough assumptions. In reference 8 the analysis takes into account the laminar sublayer, but both the original and the revised analyses given in reference 8 appear to be based on assumptions that may not be wholly correct. The revised analysis also appears to contain algebraic mistakes. In reference 9 the problem is treated only for fluids that have a Prandtl number of unity. For such fluids, T_{eff} is found to be the full stagnation temperature and

$$\frac{H}{S} = h[T_w - T_f - (\Delta T)_{ad}]$$

Many other references might be mentioned - for example, about 20 papers related to the subject are referred to in reference 10. None of them, however, seems to be entirely unquestionable, but all derive answers that, for air, are of the same order of magnitude. As will be shown later, however, the rigorous solution of the problem is not necessary for practicable heat-exchanger calculations, and only the order of magnitude of the answer is needed.

The only experimental determination of T_{eff} for high-velocity turbulent flow appears to be some measurements made with air and reported in reference 4. These measurements consisted of the determination of the temperature assumed by a wire placed axially in a nozzle. The results are shown in figure 2. Figure 2 shows that the ratio $(\Delta T)_{eff}/(\Delta T)_{ad}$ is slightly higher for turbulent flow than for laminar flow and, at least for the range of Reynolds number shown, is of the order of 0.88.

For the entrance region of a heat exchanger, in which the velocity profile in the fluid is strongly dependent upon the distance from the leading edge, no analysis has been made.

Equation (1) shows that the pumping power for a heat-transfer surface is proportional to the square of the fluid velocity. That the fluid velocity should be kept low in heat exchangers is generally realized. The discussion in the present paper of the correct fluid temperature for heat-transfer calculations with high fluid velocities is therefore primarily of only academic interest. (An exception, however, is the relatively rare case of heat transfer from an airplane wing.) In most heat exchangers, the air velocity is low enough that the effect of frictional generation of heat on the air temperature is unimportant. On the other hand, the airplane velocity is generally high enough that the adiabatic rise in temperature of the air between the free stream and the entrance to the exchanger must be taken into account. Inasmuch as the indications are, as has been previously discussed, that the effective temperature rise is given by practically the same equation as the compressional stagnation temperature rise and because the actual compressional rise generally is almost complete stagnation rise, calculations of rate of heat transfer in a heat exchanger will usually be sufficiently accurate if the temperature of the cooling air at the entrance to the heat exchanger is taken to be the full stagnation temperature of the free-stream air

$$T_{en} = T_s = T_o + (\Delta T)_{ad}$$

TABLE I
COMPARISON OF ACTUAL, STAGNATION, AND EFFECTIVE AIR
TEMPERATURES FOR VARIOUS VALUES OF V_{en} AND FOR
 $V_o = 500$ mph

V_{en} (mph)	$T_f - T_o$ (°F)	$T_s - T_o$ (°F)	$T_{eff} - T_o$ (°F)
400	16.2	45	42.1
300	28.8	45	43.4
200	37.8	45	44.3
100	43.2	45	44.8

Table I permits a comparison of the values of the actual inlet temperature T_f and the stagnation temperature T_s with the approximate values of the effective temperature T_{eff} for various entrance velocities and for a free-stream velocity of 500 miles per hour. The values shown in the table were calculated as follows:

For a velocity of 200 miles per hour in the heat-exchanger entrance, the actual fluid temperature in the exchanger entrance in °F is

$$\begin{aligned} T_{en} &= T_o + (1.8 \times 10^{-4}) [(500)^2 - (200)^2] \\ &= T_o + 37.8 \end{aligned}$$

Stagnation temperature in °F is

$$\begin{aligned} T_s &= T_o + (1.8 \times 10^{-4}) (500)^2 \\ &= T_o + 45 \end{aligned}$$

The temperature that is effective for heat transfer T_{eff} in $^{\circ}F$ is, according to the data of figure 2, of the order of

$$\begin{aligned} T_{eff} &= T_o + (1.8 \times 10^{-4}) [(500)^2 - (200)^2 + 0.9(200)^2] \\ &= T_o + 44.3 \end{aligned}$$

A comparison of the stagnation temperatures with the effective temperatures shown in the last two columns of table I shows that, at all cooling-air velocities, the difference between the two temperatures is insignificant. It therefore appears that use of full stagnation temperature as the temperature effective for heat transfer is sufficiently accurate for the calculation of heat transfer in aircraft heat exchangers.

Two further interesting facts can be mentioned. In calculating heat transfer at high altitudes, it is more desirable to use the exact heat-balance equation

$$H = Wc_p(T_{ex} - T_{en}) + \frac{W}{2Jg}(v_{ex}^2 - v_{en}^2) \quad (2)$$

than the inexact but more generally used equation

$$H = Wc_p(T_{ex} - T_{en}) \quad (3)$$

At low altitudes, the last term in equation (2) is usually negligible but, at high altitudes, may be important. If the effective temperature is taken as the stagnation temperature

$$T_{er} = T_{eff} = T_s$$

calculations in which equation (2) is used are greatly simplified. The simplification results from the fact that equation (2) can be written

$$H = Wc_p(T_{s_{ex}} - T_{s_{en}})$$

It should also be pointed out that use of the stagnation temperatures of the cooling air and the engine air instead of the actual fluid temperatures in calculations for cross-flow intercoolers in no way invalidates Nusselt's determination of the mean temperature difference in cross flow (reference 11). Nusselt's calculation of mean temperature difference is based on the actual temperatures of the fluids and on the approximate heat-balance equation (3). An inspection, however, of his analysis shows that if, in the analysis, actual temperatures are replaced by stagnation temperatures and the approximate heat-balance equation (3) is replaced by the exact equation (2), the analysis proceeds exactly as given by Nusselt and the same numerical results are obtained.

CONCLUDING REMARKS

A brief review has been given of the investigations of the effect of boundary-layer heat on rate of heat transmission. Calculations based on the results of these investigations showed that sufficient accuracy can be obtained in calculations of the rate of heat transfer in aircraft heat exchangers by using the stagnation temperature of the cooling air as its effective temperature.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va.

REFERENCES

1. Colburn, Allan P.: Heat Transfer by Natural and Forced Convection. Eng. Bull., vol. XXVI, no. 1, Res. Ser. No. 84, Eng. Exp. Station, Purdue Univ., Jan. 1942.
2. Eckert, E., and Drewitz, O.: The Heat Transfer to a Plate in Flow at High Speed. T.M. No. 1045, NACA, 1943.
3. Pohlhausen, E.: Der Wärmeaustausch zwischen festen Körpern und Flüssigkeiten mit kleiner Reibung und kleiner Wärmeleitung. Z.f.a.M.M., Bd. 1, Heft 2, 1921, pp. 115-121.
4. Eckert, E.: Temperature Recording in High-Speed Gases. T.M. No. 983, NACA, 1941.
5. Eckert, E. and Weise, W.: The Temperature of Unheated Bodies in a High-Speed Gas Stream. T.M. No. 1000, NACA, 1941.
6. Crocco, Luigi: Transmission of Heat From a Flat Plate to a Fluid Flowing at a High Velocity. T.M. No. 690, NACA, 1932.
7. Schirokow, M.: Techn. Physics, USSR, vol. 3, 1936, pp. 1020-1027.
8. Frankl, F.: Heat Transfer in the Turbulent Boundary Layer of a Compressible Gas at High Speeds; and Frankl, F., and Voischel, V.: Friction in the Turbulent Boundary Layer of a Compressible Gas at High Speeds. T.M. No. 1032, NACA, 1942.
9. von Kármán, Th., and Tsien, H. S.: Boundary Layer in Compressible Fluids. Jour. Aero. Sci., vol. 5, no. 6, April 1938, pp. 227-232.
10. Reichardt, H.: Heat Transfer through Turbulent Friction Layers. T.M. No. 1047, NACA, 1943.
11. Nusselt, Wilhelm: Eine neue Formel für den Wärmedurchgang im Kreuzstrom. Tech. Mech. u. Thermodynamik, Bd. 1, Nr. 12, Dec. 1930, pp. 417-422.

LANGLEY RESEARCH CENTER



3 1176 01354 2643